On μ and $μ^*$ Operators in Ideals Topological Spaces

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Abstract

The focus of this paper is on operations in ideal topological spaces. We look at several categorizations of Hayashi–Samuel spaces, $\mu^* - W$ sets, and β −open sets of -topology. This study also discusses decomposition.

1. Introduction

Today, the study of ideals in topological space is not a novel topic. It has been studied since the twentieth century and continues to be studied now. If a set $I \subseteq P(X)$ (power set of X) meets the finite multi-functionality, it is termed an ideal [1,3] on X. An ideal topological space (X, ρ) is a topological space with an ideal I on X. Two operators, "global function μ " [2] and "set operators μ* " [7], were crucial in the study of ideal topological spaces. In this application, the global function of $S \subseteq X$ for the ideal topological space (X, ρ, I) is defined as: $S\mu(I)$ (or merely $S\mu$) $\{x \in X : Q \cap S / \in I, Q \in \rho(x)\}$, where $\tau\ \rho(x) = \{Q \in \rho : x \in Q\}$, while μ*-operator equals $\mu^*\mu(S) = X \setminus S^c\mu$.

These two operations were overly connected to the topological space's interior and closure operators. The interior of a set S (for short, I nt(S)) can be thought of as the approximate of an open set, whereas the closure of S (denoted as Cl(S)) can be thought of as the approximate of a closed set. Moreover, it is true that $Int(S) \subseteq S \subseteq Cl(S)$. For just an ideal topological space $(X, \rho, I)$and $S \subseteq X, S\mu \subseteq Cl(S)$ and $Int(S) \subseteq \mu^*(S)$, the following holds. Nevertheless, $\mu^*(S) \nsubseteq S \nsubseteq S\mu^*$ and as a result of the ones that follow:

Take the topological space $(\mathbb{R}, \rho_U, P(\mathbb{Q}))$,when $\mathbb{R}$ represents the collection of reals, ρ_U represents the standard topology on R, and $P(\mathbb{Q})$ represents the power set of Q. Now let us assume $k \in \mathbb{Q}$. Then $\mu^*(\{k\}) = \mathbb{R}$ and $(\{k\})\mu^* = \emptyset$.

It's also worth noting that the value of a set obtained through to the joint operations of interior (resp. closure) and closure (resp. interior) is not the same as the value obtained through the joint operations of μ (resp. μ^*) and μ^* (resp. μ).

As a result, studying collections in ideal topological spaces described by the operators and will also be fascinating. The sets presented by Modak in [3] and Modak and Bandyopadhyay in [2] are noteworthy in this regard. The terms μ -set and $\mu^* - W$ set are used to describe these kinds of sets. These collections aid in the discussion of the characteristics of α-topology of $\rho^*(I)$, where $\rho^*(I)$ [9,10] is a topology derived from (X, ρ, I) and it was one of the base is $B(I, \rho) = \{N \setminus I : N \in \rho, I \in I\}$ [8]. Dontchev et al, [5], Mukherjee et al, [4] presented extensions of topological spaces in terms of ideals while, Dontchev [6] refers to them as "Hayashi–Samuel" spaces.

In this study, we show that the α-topology can be described by the collection $\mu^*(X, \rho)$ This publication also includes more characteristics of the Hayashi–Samuel space. With μ -sets and $\mu^* - W$ sets, we build further links between the generalized open sets of topological space. For the $\mu^* - W$ set, we additionally prove a deconstruction theorem. We develop a function called -function and discuss compositions of different functions in this study.

Firstly, we provide the following definition:

preopen (resp. Semi-open, semi-preopen regular open) refers to a subset S of a topological space (X, ρ). (α-open ,δ-open, β-open [10]) set if $S \subseteq Int(Cl(S))$ (resp. $S \subseteq Cl(Int(S), S \subseteq Cl(Int(Cl(S))), S = Int(Cl(S))), S \subseteq Int(Cl(Int(S)), Int(Cl(S)) \subseteq Cl(Int(S)), S \subseteq Int(Cl(Int(S))))$.) denotes the set of all POX (resp. SOX, SPOX , ROX , αOX, βOX , δOX).

2. Properties of μ^* sets

Firstly, we give the definition of the μ^*-set, and so talk about the Hayashi–Samuel space.

Definition 2.1.

Let (X, ρ, I) be an ideal topological space and $S \subseteq X$, then S is a μ^* (resp. $\mu^* - W$ set if $S \subseteq (\mu^*(S))$ (resp. $S \subseteq Cl(\mu(S))$)).

$\mu^*(X, \rho)$ (resp. $\mu(X, \rho)$) represents a set of all μ^* (resp. $\mu^* - W$ sets) in (X, ρ, I).

$\mu^* : (X, \rho, I) \to K(\rho)$ (set of all closed sets in (X, ρ)) is a predefined operator defined by $\mu^* S = ((\mu(S))^*$ and $S \subseteq X$.

Proposition 2.2.

Let (X, ρ , I) satisfies a Hayashi–Samuel space. Then

(1) $S \in \mu^*(X)$, $S \in$ ROX.

(2) $S \in \mu^*(X)$, For $S \in \mu(X, \rho)$.

(3) $S \in \mu^*(X)$ where, $S \in$ P OX and , $S \in \delta$OX.

Proof. (1) Assume S is regular open. So $S = Int(Cl(S)) \subseteq \mu(Int(Cl(\mu(S)))) \subseteq \mu(S) \subseteq ((\mu(S))^*$. Therefore, $S \in \mu^*(X)$ because, $\mu(S)$ is open and Hayashi–Samuel

(2) Put $S \in \mu(X, \rho)$. Then $S \subseteq Cl(\mu(S)) \subseteq (\mu(S))^*$.Therefore, $S \in \mu^*(X)$,because $\mu(S)$ is open and Hayashi– Samuel. Thus $A \in \Psi^*$ (X).

(3) Put $S \in$ P OX and $S \in \delta$OX . So, $S \subseteq Int(Cl(S)) \subseteq Cl(I\,nt(S))$. So $S \subseteq Cl(\mu(Int(S)))$ .Therefore, $S \subseteq Cl(\mu(S)) \subseteq (\mu(S))^*$, because S is a δ set and Hayashi–Samuel.

Following example shows that μ^*-set need not be a semi-open in general.

The following example demonstrates that μ^*-set does not have to be δOX in general.

Example 2.3.

Let $X = \{m, n, o\}$, $\rho = \{\emptyset, \{m, o\}, X\}$ and $I = \{\emptyset, \{o\}\}$. Then $\{m\}$ is μ^*-set but not δOX.

Proposition 2.4.

Let (X, ρ , I) an ideal topological space , where $I = I_k$, then $\mu^*(X, \rho) =$ POX.

Proof. To prove $\mu^*(X, \rho) \subseteq PO(X)$. Let $S \in \mu^*(X, \rho)$. So, $S \subseteq \left(\mu(S)\right)^* \subseteq$

$$Cl\left(Int\left(Cl\left(Int(Cl(S))\right)\right)\right) = Int\left(Cl(S)\right). \text{Thus, } S \in POX.$$

Proposition 2.5.

Let (X, ρ , I) an ideal topological space , where $I = I_k$, then $\mu^*(X, \rho) = \beta$OX.

Proof. It is obvious.

Proposition 2.6. Let (X, ρ , I) be an ideal topological space. Then $\mu^*(X, \rho) \subseteq \mu(X, \rho)$

Proof.

$$\text{Let } S \in \mu^*(X, \rho). \text{So, } S \subseteq \left(\mu(S)\right)^* \subseteq Cl\left(\mu(S)\right). \text{Hence, } S \in \mu(X, \rho).$$

The converse of the preceding theorem is false, as shown by the following example:

Example 2.7. Let $X = \{m, n, o, k\}$, $\rho = \{\emptyset, \{m\}, \{n, o\}, \{m, n, o\}, X\}$ and $I = \{\emptyset, \{m\}\}$. Let $S = \{m, k\}$. So, $\mu(\{m, k\}) = X \setminus (\{m, k\})^* = X \setminus (\{n, o\})^* = X \setminus \{n, o, k\} = \{m\}$. Now $(\{m\})^* = \emptyset$. So, $Cl(\{m\}) = \{m, k\}$. Hence $\{m, k\} \notin \mu^*(X, \rho)$ and $\{m, k\} \in \mu(X, \rho)$.

Proposition 2.8.

A space (X, ρ , I) is Hayashi–Samuel iff $\mu^*(X, \rho) = \mu(X, \rho)$.

Proof. Assume that $\mu^*(X, \rho) = \mu(X, \rho)$. To prove, (X, ρ , I) is Hayashi–Samuel. Since X is open in (X, ρ), $X \in \mu(X, \rho)$. Thus, $X \subseteq (\mu(X))^*$, and so $X \subseteq X^*$. Therefore, $X = X^*$, and (X, ρ , I) is Hayashi–Samuel.

Proposition 2.9.

Let (X, ρ , I)be a Hayashi– Samuel space. Then $S \in \mu^*(X, \rho)$ iff $S \in \beta O(X)$ and $S \in W(\rho)$.

Proof. Let $S \in \mu^*(X, \rho)$. Then $S \subseteq (\mu(S))^* = [Cl(\mu(S))]^* \subseteq Cl(Int(Cl(A)))$. Thus, $S \in \beta O(X)$and $S \in W(\rho)$, because the space is Hayashi– Samuel.

Vise versa, Assume that $S \in \beta O(X)$and $S \in W(\rho)$. Hence, $S \subseteq Cl(Int(Cl(S))) \subseteq Cl[\mu(Int(Cl(S)))\subseteq[\mu(Int(Cl(S)))]^* \subseteq [\mu(Cl(S))]^* = [\mu(S)]^*$. Therefore, $S \in \mu^*(X, \rho)$.

Proposition 2.10.

Let (X, ρ , I)be a Hayashi– Samuel space. Then $Cl^*(O) = Cl(O)\ \forall\, O \in \rho^*(I)$.

Proof. Let (X, ρ , I)be a Hayashi– Samuel space, $O \subseteq O^*\ O \in \rho^*(I)$, so $Cl(O) \subseteq Cl(O^*) \subseteq O^*$. Thus, $Cl(O) \cup O \subseteq Cl^*(O)$. Therefore, $Cl(O) \subseteq Cl^*(O)$.

3- Conclusion:

Operations in ideal topological spaces have been studied in several classifications of Hayashi–Samuel spaces, $\mu^* - W$ set, and β-open set from topology and some properties of $\mu^*(X, \rho)$.

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