On Topological Spaces with Pre generalized w-closed Sets

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Abstract

The goal of this paper is to present and investigate the concept of pre-generalized w-closed sets with some properties related it in the topological spaces.

1. Introduction

Bhattacharya and Lahiri (Bhattacharya and Lahiri, 1987) investigated semi-generalized closed (sg-closed) sets. In (Palaniappan and Rao 1993), Palaniappan and Rao proposed regular generalised closed (rg-closed) sets. Andrijevic (Andrijevic, 1996) developed and investigated b-open sets in 1996. Generalized pre-regular closed (briefly gpr-closed) sets were introduced by Gnanambal in (Gnanambal 1997). In topology, Sundaram and Sheik John invented the idea of w-closed sets (Sundaram and Sheik, 2000). We develop pre-generalized w-closed sets in topological spaces and investigate their basic features in this paper.

2. Preliminaries

Through this article, X refers to a topological space in which there is no separation axiom. $cl(S)$ signifies the closure of $S, int(S)$ the interior of $S, pcl(S)$ the pre-closure of S, and $bcl(S)$ the b −closure of S for any subset S of X. S^c also stands for the complement of S in X. The definitions that follow will come in handy in the next parts.

Definition 2.1.

1. Let $S \subseteq X$ Then s is
pre − open set [6] if $\mathrm{S} \subseteq \mathrm{int}(\mathrm{cl}(\mathrm{S}))$ and a pre − closed set if $\mathrm{cl}(\mathrm{int}(\mathrm{S})) \subseteq \mathrm{S}$.
2. semi-open set [8] if $\mathrm{S} \subseteq \mathrm{cl}(\mathrm{int}(\mathrm{S}))$ and a semi − closed set if $\mathrm{int}(\mathrm{cl}(\mathrm{S})) \subseteq \mathrm{S}$.
3. b − open set [14] if $\mathrm{S} \subseteq \mathrm{cl}\big(\mathrm{int}(\mathrm{S})\big) \cup \mathrm{int}\big(\mathrm{cl}(\mathrm{S})\big)$and b − closed set if $\mathrm{int}\big(\mathrm{cl}(\mathrm{S})\big) \cap \mathrm{cl}\big(\mathrm{int}(\mathrm{S})\big) \subseteq \mathrm{S}$.
4. if $\mathrm{S} = \mathrm{int}\big(\mathrm{cl}(\mathrm{S})\big)$then called a regular − open set [1]and if $\mathrm{S} = \mathrm{cl}\big(\mathrm{int}(\mathrm{S})\big)$ then called a regular − closed set.

Definition 2.2.[11]

Let $S \subseteq X$ Then S is a w-open set if $\mathrm{int}(\mathrm{cl}(\mathrm{S})) \subseteq \mathrm{S} \subseteq \mathrm{cl}(\mathrm{int}(\mathrm{S}))$.

Definition 2.3.

Let $\subseteq X$.

1. if $cl(S) \subseteq M$ whenever $S \subseteq M$ *and* M is open in X. Then S is generalized closed set (for short, $g-$ closed) [15].
2. if $pcl(S) \subseteq M$ whenever $S \subseteq M$ *and* M is regular-open in X. Then S is generalized pre-regular closed set (for short, $g$pr − closed)[6].
3. if $cl(S) \subseteq M$ whenever $S \subseteq M$ *and* M is regular-open in X. Then S is regular-generalized closed set for short, rg − closed) [10].
4. if $cl(int(S)) \subseteq M$ whenever $S \subseteq M$ *and* M is regular-open in X. Then S is weakly generalized closed set (for short,, rw$g$ − closed)[11].
5. if $bcl(S) \subseteq M$ whenever $S \subseteq M$ *and* M is regular-open in X. Then S is regular generalized b-closed set (for short, rgb − closed)[7].
6. if $cl(S) \subseteq M$ whenever $S \subseteq M$ *and* M is semi-open in X. Then S is weakly closed (for short,w-closed) set [11].
7. if $scl(S) \subseteq M$ whenever $S \subseteq M$ *and* M is semi-open in X. Then S is semi-generalized closed set (for short , sg − closed)[3].
8. if $bcl(S) \subseteq M$ whenever $S \subseteq M$ *and* M is semi-open in X. Then S is semi-generalized b-closed set (for short, sgb − closed) [4].
9. if $scl(S) \subseteq M$ whenever $S \subseteq M$ *and* M is open in X. Then S is generalized semi-closed (for short, gs − closed set)[5].
10. if $cl(S) \subseteq M$ whenever $S \subseteq M$ *and* M is w-open. Then S is generalized w-closed set (for short, gw-closed set) [12].
11. If $cl(S) \subseteq M$ whenever $S \subseteq M$ *and* M is gs −open in X. Then S is gs^* − closed set [9].

3. β-GENERALIZED W-CLOSED SETS

Pre-generalized w-closed sets in topological spaces are introduced in this section. We also go over some of their fundamental characteristics

Definition 3.1.

Let $S \subseteq X$ Then S is β-generalized w-closed (briefly, βgw- closed) if $\beta\, cl(S) \subseteq M$ whenever $S \subseteq M \text{ and } M$ is w-clopen.

Proposition 3.2.

Let $S \subseteq X$ Then. Then every set S which is w-closed set is βgw-closed.

Proof: Let S be a w-closed set. Let M be a w-open set containing S. Since every w-open set is β-open, so M is β-open. Hence, $cl(S) \subseteq M$. Now, since $\beta cl(S) \subseteq cl(S)$, then $\beta cl(S) \subseteq M$. Thus, S is βgw-closed.

As the following example shows, the converse of Proposition 3.2 does not have to be true.

Example 3.3.

Let $X = \{h,k,l,m,n\}$ and $\rho = \{\phi,\{h,k\},\{l,m\},\{h,k,l,m\},X\}$ is a topology on X. Thus $\{h,k,l\}$ is βgw -closed but not w-closed.

Proposition 3.4. Let (X,ρ) be a T.S. Then every closed set is βgw -closed.

Proof: Let S be closed set. `So S is w-closed because each closed set is w-closed. Hence, S is βgw -closed (Proposition 3.2).

As the following example shows, the converse of Proposition 3.4 does not have to be true.

Example 3.5.

Let $X = \{h,k,l,m\}$ and $\rho = \{\phi,\{h\},\{k\},\{h,k\},\{k,l\},\{h,k,l\},\{h,k,m\},X\}$ is a topology on X. Thus $\{k,l\}$ is βgw -closed but not closed.

Proposition 3.6.

Let (X,ρ) be a T.S. Then every gs^*-closed set is βgw -closed.

Proof: Let S be gs^*-closed set. So $cl(S) \subseteq Q$ where $S \subseteq Q \text{ and } Q$ is gs-open. Let M be a w-open set. Now, since every w-open set is gs-open, so M is gs-open. Hence $cl(S) \subseteq M$. Since $\beta cl(S) \subseteq cl(S)$, so $pcl(S) \subseteq M$. Therefore S is βgw -closed.

As the following example shows, the converse of Proposition 3.6 does not have to be true.

Example 3.7.

Let $X = \{h,k,l,m\}$ and $\rho = \{\phi,\{h\},\{k\},\{h,k\},\{h,k,l\},\{h,k,m\},X\}$ is a topology on X.Thus, $\{h,k\}$ is βgw -closed but not gs^*-closed.

Proposition 3.8.

Let (X,ρ) be a T.S. Then every regular closed set is βgw-closed.

Proof: obvious

As the following example shows, the converse of Proposition 3.8 does not have to be true.

Example 3.9.

Recall Example 3.7, we get $\{h,k,l\}$ is βgw -closed but not regular closed.

Proposition 3.10.

Let (X,ρ) be a T.S. Then every gs^*-closed set is βgw -closed.

Proof: Let S be a gs^*-closed set. Set M be w-open set. Then $cl(S) \subseteq M$. Now, since $\beta cl(S) \subseteq cl(S)$, so $\beta cl(S) \subseteq M$. Thus, S is βgw -closed.

As the following example shows, the converse of Proposition 3.10 does not have to be true.

Example 3.11.

Let $X = \{h,k,l,m\}$ and $\rho = \{\phi,\{h\},\{k\},\{h,k\},\{h,l\},\{h,k,l\},X\}$ is a topology on X. Then $\{h,k\}$ is βgw -closed but not gs^*-closed.

Proposition 3.12.

Let (X,ρ) be a T.S. Then every βgw -closed set is gpr-closed.

Proof: Let S be βgw -closed set. Set Q be a regular open set. So Q is w-open because every regular open set is w-open. Hence, $\beta cl(S) \subseteq Q$. Therefore, S is gpr -closed.

As the following example shows, the converse of Proposition 3.12 does not have to be true.

Example 3.13.

Let $X = \{h,k,l,m,n\}$ and $\rho = \{\phi,\{h\},\{m\},\{n\},\{h,m\},\{h,n\},\{m,n\},\{h,m,n\},X\}$ is a topology on X. Thus, $\{h,n\}$ is gpr -closed but not βgw -closed.

Proposition 3.14.
Let (X,ρ) be a T.S. Then every βgw -closed set is rgb-closed.
Proof: Let S be a βgw -closed set. Then $\beta cl(S) \subseteq M$ where $S \subseteq M$ and M is w-open. Set Q be a regular open set. So, M is w-open. hence, $\beta cl(S) \subseteq Q$. And so , $bcl(S) \subseteq \beta cl(S)$ because every pre-closed set is b-closed. Thus, $bcl(S) \subseteq U$. Therefore, S is rgb -closed.
As the following example shows, the converse of Proposition 3.14 does not have to be true.
Example 3.15.
Let $X = \{h,k,l,m\}$ and $\rho = \{\phi,\{h\},\{m\},\{h,k\},\{k,l\},\{h,k,l\},\{h,k,m\},X\}$ is a topology on X.Then $\{m\}$ is rgb-closed but not βgw -closed.
Proposition 3.16. Let (X,ρ) be a T.S. If S is βgw -closed set s.t, $S \subseteq J \subseteq \beta cl(S)$, then J is βgw -closed set in X.
Proof: Let M be w-open set. So, $S \subseteq M$. Since A is βgw -closed, so $\beta cl(S) \subseteq M$. Hence $\beta cl(J) \subseteq M$, because $J \subseteq \beta cl(S)$. Thus, J is a βgw -closed set in X.
Proposition 3.17.
Let (X,ρ) be a T.S. Then all the subsets of X are βgw -closed if $\emptyset$ and X are the only w-open sets.
Proof: Let $S \subseteq X$. If $S = \emptyset$, then S is βgw -closed. If $A \neq \emptyset$, then X is only w- open set. So, $\beta cl(S) \subseteq X$. Therefore, S is βgw -closed.
As the following example shows, the converse of Proposition 3.16 does not have to be true.
Example 3.18.
Let $X = \{h,k,l,m\}$ and $\rho = \{\phi,\{h\},\{k\},\{l\},\{h,k\},\{h,l\},\{k,l\},\{l,m\},\{h,k,l\},\{k,l,m\},\{h,l,m\},X\}$ is a topology on X. Thus, all the subsets of X are βgw -closed but the w-open sets are $\phi,\{h\},\{k\},\{l\},\{h,k\},\{h,l\},\{k,l\},\{l,m\},\{h,k,l\},\{k,l,m\},\{h,l,m\},X$.
Proposition 3.19.
Let $S \subseteq X$. Then S is βgw -closed iff $\forall$ $S \subseteq M$ and M is w-open, $\exists$ pre-closed set V s.t, $S \subseteq V \subseteq M$.
Proof: Assume that S is βgw -closed and $S \subseteq M$ and M is w-open. Then $\beta cl(S) \subseteq M$. Set $V = \beta cl(S)$, so, $S \subseteq V \subseteq M$. Vise versa, suppose that M is w-open set. Thus, $\exists$ pre-closed set V s.t, $S \subseteq V \subseteq M$ (by hypothesis). Since $\beta cl(A)$ is pre-closed set, so, $\beta cl(S) \subseteq V$. Hence, $\beta cl(S) \subseteq M$. Thus, S is βgw -closed.
Proposition 3.20.
Let (X,ρ) be a T.S, and let $S \subseteq X$. If S is regular open and βgw -closed, then S is both β-open and β-closed.
Proof: Suppose that S is regular open and βgw-closed. So, $\beta cl(S) \subseteq S$, because every regular open set is w- open, Hence $S = \beta cl(S)$. And so, S is β-closed. Also, S is β-open, because S is regular open. Therefore, S is both β-open and β-closed.

Conclusion

We presented βgw -closed sets in topological spaces and looked at several of their basic features in this study. In addition, the relation among βgw -closed sets and several generalized sets in topological spaces has been investigated.

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