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(Study and Solve Some of the Characteristics Of Differential Sobolev type equations)

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Abstract:

In this paper we study the properties of a pseudodifferential operator

$$L=(\lambda + \Delta)$$

Where is the Laplace - Beltrami operator in the space of differentiable sobolev defined on smooth compact oriental connected Riemannian manifolds without boundary. To secure one's object, you need to solve the following tasks :

-introduce Riemannian manifolds without boundary

-introduce pseudodifferential operators over these differential k - forms, for example, Laplace - Beltrami plus lower coefficients;

-Investigate the properties of the obtained operators.

The results obtained were based on the sobolev on the splitting of the space of differential forms into a direct sum of subspaces.

1- Introduction

Various mathematical models based on Sobolev-type equations have the form

$$Lu^{\circ} = M \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \quad . \qquad (1)$$

with an irreversible operator L at the derivative [1]. It should be noted that the use the following are carried out both for abstract equations and for specific statements of this kind [2] .

Qualitative and numerical studies available solutions of these equations in a variety of settings. We have been be interested to solve one of the aspects of the solvability of initial-boundary value problems in space differential k-forms defined on a Riemannian manifold without boundary, for example, the Cauchy problem

$$u(x) = u_x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

for equations of the form (1), namely, what is required in the theory of relative opera

splitting the domain of an operator L∈£(ɪɪ,F) into a direct sum of subspaces

$$U = \; U^0 \oplus U^1. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

For some spaces, these splittings are natural, while for others you have to try to get them. We are talking here about differentiable k-forms defined on smooth compact orientable connected Riemannian manifolds without boundary , studies of the properties of pseudodifferential operators in spaces of differential forms began. One of the most significant among these operators is the Laplace – Beltrami operator on differential forms, which, up to sign, generalizes the Laplace operator. These studies are quite effective due to the well-known properties of this operator. The transition to Banach spaces is possible thanks to the results of Leng.[4]

The purpose of this work is a study of the properties of the pseudodifferential operator

$$L=\lambda+\Delta\ ,$$

Where Δ is the Laplace – Beltrami operator in the space of differentiable k-forms defined on smooth compact orientable connected Riemannian manifolds :without boundary. To achieve this goal, the following tasks must be solved

introduce into consideration Riemannian manifolds without boundary :

construct differential k-forms on Riemannian manifolds without boundary;

introduce pseudodifferential operators over these differential k-forms, for example, Laplace – Beltrami plus lower-order coefficients; to investigate the properties of the obtained operators.

In the introduction, the problem is formulated and its connection with mathematical models of the Sobolev type is described. The first section contains preliminary information from the theory of relatively bounded operators.

Definition 1:

A pseudodifferential operator is a natural extension of the notion of a partial differential operator. Consider the differential operator Q of degree m, defined on the set of m times differentiable functions of n variables :

$$Q[\omega](x)\ =\sum_{|k1+\cdots+k\_n\ |\le m} ak_{1+\cdots+k_n}\ .\ \frac{\partial k_{1\omega}}{\partial x_1^{k1}}(x),$$

Where $a_{\alpha=\,const,}\forall\alpha$.

Associate with Q the polynomial

$$g(y)=\sum_{k\_1+\cdots+k\_n\ |\le m} ak_{1+\cdots+k_n(iy)^{|k1+\cdots+kn}}$$

ich is called the symbol Q.

2. Relatively p-bounded operators

Let the operators U and F be Banach spaces, the operators

M $\epsilon\mathscr{L}$(u,f) A bunch of $P^L(M) = \{\mu \in \mathbb{c}\,; (\mu L - M)^{-1}\}\epsilon\mathcal{L}(F, u)$ … … . . (4)

is called the L-resolvent set of the operator M.

The set $\sigma^L(M) = \mathbb{C} \setminus P^L(M)$ *is* called the L-spectrum of the operator M.

If $P^L(M) \neq \emptyset$, then, we can define an operator function

$(\mu\mathcal{L} - M)^{-1}, R_\mu^L(M) = (\mu\mathcal{L} - M)^{-1}L, \mathcal{L}_\mu^L(M) = (\mu\mathcal{L} - M)^{-1}$ … … . . . (5)

which are called, respectively, the $\mathcal{L}$-resolution, Right resolution of L, Left L-resolvent of M. In the case where there is an operator $L^{-1} \in (\mathfrak{F}, \mathfrak{A})$, L-resolution, right L-resolvent and left L- resolvent of M coincides with the resolvent of the operator M, ML^{-1} and $L^{-1}M$ accordingly.

The operator M is called the determinant factor for L (in short, (L, σ) – (bounded), if

$$\exists a \in R_+, \forall\mu \in \mathrm{C}(|\mu| > a).$$

Let $P^L(M) \neq \emptyset$. Equation (1) reduce to a pair of equivalent him equations .

$$R_\mu^L(M)u^\circ = (\mu L - M)^{-1}Mu, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

$$L_\mu^L(M)F^\circ = M = (\mu L - M)^{-1}f \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

where $\mu \in P^L(M)$.

Both equations can be understood according to [1] as concrete interpretations of the equation

$$Xc^{\circ}=Yc \text{ ,...(8)}$$

where the operators X, Y ∈ $\mathcal{L}$ (W), a W is Banach space.

<u>Lemma1 :</u>

Let the operator M ($\mathcal{L}$, σ) -bounded, and the contour : Γ = (μ ∈ C :|μ|=r>a) . Then the operators P: $u \to \mathfrak{U}$ and Q: $\mathfrak{F} \to \mathfrak{F}$ defined in terms of integrals of the type ϕ.

$$P = \frac{1}{2\pi i}\int_{\Gamma} R_{\mu}^{L}(M) d\mu \ ,$$

$$Q = \frac{1}{2\pi i}\int_{\Gamma} L_{\mu}^{L}(M) d\mu \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

We put

$$\mathfrak{U}^{0} = \ker P \ , \ imP = \mathfrak{U}^{1} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

$$\mathfrak{F} = \ker Q \ , \mathfrak{F}^{1} = \mathrm{im}Q \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

denote by L_k (Mk) the restriction of the operator L (M) to $\mathfrak{U}^{K}$, $K = 0,1$.

$$E = M_0^{-1}L_0 \ , E \in \mathcal{L}(\mathfrak{U}^0), \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

and operator

$$S = L_1^{-1}M_1, S \epsilon\ \mathcal{L}(\mathfrak{U}^1) \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

by means of which the L-resolution of the operator M expands in the ring | μ |> a in the Laurent series

$(\mu L-M)^{-1} = -\sum_{k=0}^{\infty} \mu^k E^k M_0^{-1} (\mathbb{I} - Q) + \sum_{K=0}^{\infty} \mu^{-K} S^{K-1} \ell_1^{-1} Q \ldots$ (14)

Here $E^0 = \mathbb{I}, if\ E \neq 0, and\ s^0 \neq 0,$

where the identity operators are defined on the spaces $\mathfrak{A}^0$ and $\mathfrak{A}^1$, respectively .

Definition2 :

Let the operator M be (L, σ) –bounded The point ∞ is called

(i) a removable singular point of the L-resolution of the operator M if E ≡ 0

(ii) a pole of order p of the L-resolution of the operator M if

$$E^P \neq 0, E^{P+1} \equiv 0 ;$$

(iii) an essentially singular point of the L-resolution of the operator M, if a$E^K \neq 0$ for any

k ∈ {0} ∪ N. In what follows, we agree to call the removable singular point a pole of order zero .

Definition 3 :

An operator M is called (L, p) -bounded if the operator M is (L, σ) -bounded and ∞ is a pole of

In what follows, we agree on any vector $\varphi \epsilon\ ker\{0\}\ is\ called\ an\ eigenvector\ of\ the\ operator\ L$. An ordered set $\{\varphi_1, \varphi_2, \ldots\} \subset \mathfrak{A}$ is called a chain M of associated vectors of the eigenvector φ_0 if :

$$L\varphi + 1 = M\varphi_q, \varphi_q \notin \ker L , q = 0,1 \ldots\ldots$$

the chain is finite if there exists an -adjoint vector φ_p that either $\varphi_p \notin$domM, or M$\varphi_p \notin imL$.

In particular, the eigenvectorφ_0 does not have M- attached vectors, if either $\varphi_0 \notin imL$.

The power of a finite chain is called its length. When the chain is endless, then we say that it has an infinite length. The linear span of all eigenvectors and M-associated vectors of the operator L is called the M root lineal operator L.

If the M-root lineal is closed, then it is called the M-root space of the operator L .

Theorem 1 :

Let the operator be L-Fredholm. Then the following conditions are equivalent

(i) Operator M (L, p) -bounded, p ∈ {0} ∪ ℕ.

(ii) The length of any chain of M-adjoint vectors of the operator L does not exceed p.

Definition 4:

Let X and Y be vector spaces. A linear operator T: X → Y is called Fredholm if its kernel and cokernel, are finite-dimensional. If X and Y are Banach spaces,

then Fredholm operators from X and Y are assumed to be bounded by default .

Definition 5:

An operator L is called Fredholm if its index ind L = 0 .

Let X and Y be Banach spaces. A bounded linear operator T: X → Y is Fredholm if and only if its adjoint operator T ∗: Y ∗ → X ∗ is Fredholm. Wherein

(i) ind T^* = −ind T ;

(ii) ind T = dim Ker T- dim Ker T^*)

Theorem 2:

Let the operator *M* (L, p) be bounded. Then there exist analytic resolving groups of equations (3) and (4) representable by the Dunford - Taylor integrals

$$U^t = \frac{1}{2\pi i} \int_r R_\mu^L (M) e^{\mu t} d\mu, \ldots\ldots\ldots\ldots\ldots\ldots..(15)$$

$$F^t = \frac{1}{2\pi i} \int_r L_\mu^L e^{\mu t} d\mu \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(16)$$

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